AP Calculus AB
Section 3.6 – Implicit Derivatives Worksheet Answer Key

Name: ______________________
Date: _____________ Block: ___

Section 1: Determine the derivative of the function, y'.

1.) $x^2 - y^2 = 9$

$2x - 2y \cdot y' = 0$
$2x - 2yy' = 0$
$-2yy' = -2x$
$y' = \frac{x}{y}$

2.) $xy = 4$

$x \cdot y' + y \cdot (1) = 0$
$xy' + y = 0$
$xy' = -y$
$y' = -\frac{y}{x}$

3.) $xy^2 - x + 16 = 0$

$(x \cdot 2y \cdot y' + y^2 \cdot (1)) - 1 + 0 = 0$
$2xyy' + y^2 - 1 = 0$
$2xyy' = 1 - y^2$
$y' = \frac{1-y^2}{2xy}$

4.) $4x^3 + 11xy^2 - 2y^3 = 0$

$12x^2 + \left(11x \cdot 2y \cdot y' + y^2 \cdot (11)\right) - 6y^2 \cdot y' = 0$
$12x^2 + 22xyy' + 11y^2 - 6y^2y' = 0$
$22xyy' - 6y^2y' = -12x^2 - 11y^2$
$y'(22xy - 6y^2) = -12x^2 - 11y^2$
$y' = \frac{-12x^2-11y^2}{22xy-6y^2}$

5.) $6x - \sqrt{2xy} + xy^3 = y^2$

$$6x - (2xy)^{\frac{1}{2}} + xy^3 = y^2$$
$$6 - \tfrac{1}{2}(2xy)^{-\frac{1}{2}} \cdot \left(2x \cdot y' + y \cdot (2)\right) + \left(x \cdot 3y^2 \cdot y' + y^3 \cdot (1)\right) = 2y \cdot y'$$
$$6 - \frac{1}{2\sqrt{2xy}}(2xy' + 2y) + 3xy^2y' + y^3 = 2yy'$$
$$6 - \frac{xy'}{\sqrt{2xy}} - \frac{y}{\sqrt{2xy}} + 3xy^2y' + y^3 = 2yy'$$
$$-\frac{xy'}{\sqrt{2xy}} + 3xy^2y' - 2yy' = \frac{y}{\sqrt{2xy}} - y^3 - 6$$
$$y'\left(-\frac{x}{\sqrt{2xy}} + 3xy^2 - 2y\right) = \frac{y}{\sqrt{2xy}} - y^3 - 6$$
$$y' = \frac{y - y^3\sqrt{2xy} - 6\sqrt{2xy}}{-x + 3xy^2\sqrt{2xy} - 2y\sqrt{2xy}}$$

6.) $xy + \sin y = x^2$

$$x \cdot y' + y \cdot (1) + \cos(y) \cdot y' = 2x$$
$$xy' + y + \cos(y)\, y' = 2x$$
$$xy' + \cos(y)\, y' = 2x - y$$
$$y'(x + \cos(y)) = 2x - y$$
$$y' = \frac{2x - y}{x + \cos(y)}$$

7.) $x^3y + y^3x = 10$

$$\left(x^3 \cdot y' + y \cdot (3x^2)\right) + \left(y^3 \cdot (1) + x \cdot 3y^2 \cdot y'\right) = 0$$
$$x^3y' + 3x^2y + y^3 + 3xy^2y' = 0$$
$$x^3y' + 3xy^2y' = -3x^2y - y^3$$
$$y'(x^3 + 3xy^2) = -3x^2y - y^3$$
$$y' = \frac{-3x^2y - y^3}{x^3 + 3xy^2}$$

8.) $x^2y^2 + 3xy = 10y$

$$\left(x^2 \cdot 2y \cdot y' + y^2 \cdot (2x)\right) + \left(3x \cdot y' + y \cdot (3)\right) = 10y'$$
$$2x^2yy' + 2xy^2 + 3xy' + 3y = 10y'$$
$$2x^2yy' + 3xy' - 10y' = -2xy^2 - 3y$$
$$y'(2x^2y + 3x - 10) = -2xy^2 - 3y$$
$$y' = \frac{-2xy^2 - 3y}{2x^2y + 3x - 10}$$

Section 2: Find the slope and the equation of the tangent line at the given point.

9.) $\sin(xy) = y$ at $\left(\frac{\pi}{2}, 1\right)$

$\cos(xy) \cdot \left(x \cdot y' + y \cdot (1)\right) = y'$
$xy' \cos(xy) + y \cos(xy) = y'$
$xy' \cos(xy) - y' = -y \cos(xy)$
$y'(x \cos(xy) - 1) = -y \cos(xy)$
$y' = -\frac{y \cos(xy)}{x \cos(xy) - 1}$

$$y' = -\frac{(1)\cos\left(\left(\frac{\pi}{2}\right)(1)\right)}{\left(\frac{\pi}{2}\right)\cos\left(\left(\frac{\pi}{2}\right)(1)\right) - 1} = -\frac{(1)(0)}{\left(\frac{\pi}{2}\right)(0) - 1} = 0$$

$y - 1 = 0(x - \frac{\pi}{2})$
$y = 1$

10.) $y + \cos(xy^2) + 3x^2 = 4$ at $(1, 0)$

$y' - \sin(xy^2) \cdot \left(x \cdot 2y \cdot y' + y^2 \cdot (1)\right) + 6x = 0$
$y' - \sin(xy^2)\,(2xyy' + y^2) + 6x = 0$
$y' - 2xyy' \sin(xy^2) - y^2 \sin(xy^2) + 6x = 0$
$y' - 2xyy' \sin(xy^2) = y^2 \sin(xy^2) - 6x$
$y'(1 - 2xy \sin(xy^2)) = y^2 \sin(xy^2) - 6x$
$y' = \frac{y^2 \sin(xy^2) - 6x}{1 - 2xy \sin(xy^2)}$

$$y' = \frac{(0)^2 \sin\left((1)(0)^2\right) - 6(1)}{1 - 2(1)(0) \sin\left((1)(0)^2\right)} = \frac{0 - 6}{1 - 0} = -6$$

$y - 0 = -6(x - 1)$
$y = -6(x - 1)$